The cyclical behavior of optimal bank capital

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Abstract

This paper presents a dynamic model of optimal bank capital in which the bank optimizes over costs associated with failure, holding capital, and flows of external capital. The solution to the infinite-horizon stochastic optimization problem is related to period-by-period value at risk (var) in which the optimal probability of failure is endogenously determined. Over a cycle, var is positively correlated with optimal flows of external capital, but negatively correlated with optimal net changes in capital and the optimal level of total capital. Analysis of this pattern suggests that a regulatory minimum requirement based on var, if binding, is likely to be procyclical. The model points to several ways of reducing this problem. For example, a var-based requirement makes more sense if it is applied to external capital flows than if it is applied to the total level of capital. US commercial bank data since 1984 are generally consistent with the model.
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1. Introduction

This paper examines the optimal behavior of a bank with regard to capital when the bank is exposed to stochastic losses with a cyclical predictable component, and investigates the extent to which bank capital requirements based on value at risk

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(var) are likely to exacerbate an economic or financial cycle. For instance, would such requirements be binding on prudent banks in economic recessions, leading to a reduction in lending as the economy is slowing down, and non-binding in economic expansions, perhaps encouraging excessive lending? Earlier research has inquired whether the 1988 Basel Accord, in which minimum capital requirements are somewhat responsive to risk, contributed to an economic slowdown – a "credit crunch" – in the United States in the early 1990s. [1]

With the introduction of new risk-sensitive Basel requirements in 2001 (Basel Committee, 2001a), concerns about the so-called procyclicality of capital requirements have again surfaced. For instance, Bank for International Settlements General Manager Andrew Crockett (2000) has warned that: "Indicators of risk tend to be at their lowest at or close to the peak of the financial cycle, i.e., just at the point where, with hindsight, we can see that risk was greatest."

The new Basel requirements are not based on var. They rely on internal or external credit ratings to assign risk weights to instruments in banks' portfolios. Nevertheless, the Basel Committee (2001b) "believes that improvements in risk measurement and management will pave the way to an approach that uses full credit models as a basis for regulatory capital purposes". Therefore, it is not too early to consider the consequences of using credit risk models to formulate capital requirements, and it is likely that those requirements would make use of model-based var or some component thereof (see Basel Committee, 1999). [2]

To examine the issue of the procyclicality of var-based requirements, the main strategy of this paper is to construct a formal model of how a forward-looking bank with rational expectations would choose its optimal level of capital in a stochastic dynamic setting. We also consider regulatory objectives and the behavior of an optimizing bank if it is subject to potentially binding var-based requirements.

The bank in the model faces three types of capital-related costs: the cost of holding capital, the cost of failure, and the cost of net changes in external capital. The objective of the bank is to minimize a function of the three types of costs over an infinite horizon, given some dynamic identities. The bank is assumed to have rational expectations, and it selects a stream of net external capital flows over the infinite horizon. [3] The solution allows for the expression of the current level of capital and the current external capital flow as functions of current and expected future net losses to the bank.

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[1] The conclusions of this research are mixed. See, e.g., Bernanke and Lown (1991), Lown and Peristiani (1996) and Jackson et al. (1999).

[2] Note also that Gordy (2000) argues that a mapping may be constructed between var and the minimum requirements in the new Basel Accord (Basel Committee, 2001a), and that the mapping reconciles the two approaches under certain conditions. Thus, the analysis of this paper could be useful in the implementation of the new Accord, though differences between the Accord and var must be carefully considered. Ervin and Wilde (2001), European Central Bank (2001) and Jokivuolle and Peura (2001) have expressed concerns about the cyclical implications of the new Basel Accord, as have policymakers such as Greenspan (2001) and Clementi (2001).

[3] Estrella (2003) discusses cases in which expectations are non-rational, e.g., they are based on rules of thumb.

To investigate cyclical behavior, losses are then assumed to follow a second-order autoregressive process with complex roots, so that expected future losses cycle around the unconditional mean of the distribution of losses. [4] This assumption makes it possible to calculate all expected future losses and to solve explicitly for the current level and flow of capital in terms of observable losses. The theoretical implications of these results are used to examine the relationship between var-based requirements and the unconstrained optimum path of capital. In addition, some theoretical implications of the model are used to test its empirical plausibility when confronted with data for US banks since 1984.

In the context of the basic model of this paper, the bank is a prudent optimizer and there are no externalities, so there is no formal need for regulatory or supervisory intervention. Nevertheless, under certain circumstances, authorities might issue minimum capital requirements as a means of correcting market imperfections or simply of identifying any banks that stray away from optimizing behavior. Thus, capital requirements could be introduced as benchmarks to verify that banks are holding capital levels that are consistent with regulatory objectives over the cycle. [5] We consider what forms those benchmarks could take and, in particular, whether they may be based on var. In addition, we consider how optimizing behavior would be altered if a var-based minimum were explicitly imposed.

The theoretical findings of the paper show that optimal flows of external capital are positively correlated over the cycle with the var measure, but that optimal net changes in capital and the optimal level of total capital are negatively correlated with var. These results suggest that a regulatory minimum requirement based on var, if binding, is likely to be procyclical. Of course, procyclicality is a macroeconomic phenomenon, whereas the cyclical behavior investigated here is microeconomic. However, if the cyclical pattern of bank losses is driven by, say, a macroeconomic business cycle, the pattern is bound to affect banks with similar timing and directionality. In the aggregate, procyclicality would ensue. By examining the causes of procyclicality, the model also suggests several ways of dealing with this potential problem.

These results diverge in some ways from the conventional wisdom regarding capital and risk because the standard approach to optimal capital is essentially static and not sufficiently forward-looking. The model of this paper takes explicit account of stocks and flows of capital over time and incorporates dynamic adjustment costs that are likely to exist in practice.

Section 2 of the paper presents the model and its solution in general terms and discusses the possible use of var to achieve regulatory objectives. Section 3 presents the central results of the paper. It introduces a cyclical process for losses and examines its implications for the cyclical behavior of optimal capital levels and flows. In

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[4] Theoretical analyses of dynamic capital structure models have frequently assumed that asset values follow geometric Brownian motion processes, which cannot produce cyclical behavior as defined here. See, for example, Fischer et al. (1989) and Leland (1994, 1998).

[5] Other reasons have been proposed for the existence of capital requirements. Very useful surveys are Berger et al. (1995) and Santos (2001).

addition, Section 3 provides numerical examples to illustrate the properties of the model and the relationships among var, net income, and capital stocks and flows. Some empirical evidence, based on call report data for FDIC-insured US commercial banks, is provided in Section 4. Finally, Section 5 discusses policy implications.

2. Model description and general solution

The question of this paper is dynamic in a very essential way. We would like to examine the cyclical behavior of optimal capital for a bank, where a cycle is defined as a particular predictable pattern that unfolds over time. Since the pattern is predictable, it can and should be anticipated in the bank's decision-making, in particular in those decisions affecting capital. [6] A static model may be able to identify some features of these optimal decisions, but it is necessarily myopic and any conclusions about dynamics are bound to be ad hoc.

Some of the issues addressed by the model have been raised earlier in the literature, but by and large the analysis has been based on essentially static models. For instance, the key distinction between flows of external and internal capital has been made in papers by Froot et al. (1993), and Froot and Stein (1998). These papers develop a three-period model of bank capital with costs analogous to those of this paper, including costs of capital, failure, and adjustment. Since the model is moderately dynamic – it has three periods – it is possible to make distinctions between capital stocks and flows, and to speak of costs of adjustment, but not to analyze cyclical patterns.

Winter (1994) and Cummins and Danzon (1997) construct dynamic models of insurance company balance sheets in which capital plays an important role. They include costs analogous to those of this paper in a static theoretical model of an insurance company, which is then embedded in a dynamic empirical model. The authors draw important distinctions between stocks and flows of internal and external capital.

Some earlier papers have raised the issue of the cyclical behavior of capital explicitly, albeit within the framework of static models. These include the property–liability insurance model of Winter (1991) and the banking models of Blum and Hellwig (1995), Heid (2000), and Hoggarth and Saporta (2001). [7]

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[6] Note that the fact that there is a predictable cyclical pattern does not imply that bank profits and capital are totally predictable, nor that the predictable cyclical pattern is simple. In Section 3, we focus on impulse response functions, which illustrate the relatively simple effects of an isolated stochastic shock to losses. However, when shocks occur every period, forecasts of the predictable pattern must be updated every period as well and the pattern may be much more complex.

[7] The closest precedents for the formal dynamic structure of the present model are not in the financial capital literature, but in the literature on business inventories (for instance, Blanchard, 1983; Blanchard and Fischer, 1989).

2.1. Characteristics of the bank

We consider a bank operating in a discrete-time infinite-horizon environment. The bank's stylized balance sheet at the beginning of period t is given by

$$F_t + V_t = D_t + I_t, \tag{1}$$

where F = value of safe asset, V = value of risky asset, D = deposits, and I = initial capital (beginning of period).

Bank assets may be invested in the safe instrument, say government bonds, which has a deterministic yield, and in the risky asset, say loans, whose return is stochastic. Bank liabilities consist of deposits only. At the start of each period, the bank has some capital carried over from the previous period (K_{t-1}) and it raises a net amount (R_t) from external sources. We call the sum of these two terms "initial capital" ($I_t = K_{t-1} + R_t$).

The bank's income statement is

$$r_t^F F_t + r_t^V V_t + R_t = r_t^D D_t + \Delta K_t, \tag{2}$$

where r_t^F and r_t^D are deterministic returns and r_t^V is a stochastic return on the risky asset. Define net losses as

$$L_t = r_t^D D_t - r_t^F F_t - r_t^V V_t. \tag{3}$$

Net losses are simply the negative of net income or profits for the bank, but working with losses is more convenient because they relate more clearly to var. At the end of the period, net losses sustained are revealed, and end-of-period capital is given by the dynamic identity

$$K_t = K_{t-1} + R_t - L_t, \tag{4}$$

which is implicit in the income statement (2).

The return on the risky asset is modeled as

$$r_t^V = E_t r_t^V - \eta_t, \tag{5}$$

where η_t is a random shock with $E_t \eta_t = 0$. Then

$$E_t L_t = r_t^D D_t - r_t^F F_t - (E_t r_t^V) V_t \tag{6}$$

and

$$L_t = E_t L_t + u_t \tag{7}$$

with

$$u_t = \eta_t V_t. \tag{8}$$

From (8), we see that V_t determines the scale of the risk embodied in u_t.

Losses are known with certainty only at the end of the period. However, at the start of the period the bank is assumed to know the probability distribution of the period's losses, which is a function of V_t and of the random variable η_t. The timing of the capital and loss variables and of the resolution of uncertainty is summarized in Table 1.

Table 1
Timing conventions for period t

Time $t - 1$ (beginning of period t)	Time t (end of period t)
Assess how much capital is available (K_{t-1})	Losses for period become known (L_t)
Decide how much capital to raise (R_t)	End of period capital is determined (K_t)
Initial available capital $(I_t = K_{t-1} + R_t)$	
Stochastic distribution of losses $(F(u_t))$	Value of u_t known
Expected losses $(E_t L_t)$	

Note that capital here represents equity capital only. Including other elements of bank capital is possible, but it would complicate the dynamics without adding much to the intuition derived from the model. Net external capital raised is a combination of inflows (new external capital) and outflows (dividends, stock buybacks). In normal circumstances, these flows are dominated by dividend payments in the case of US banks, as we observe in Section 4.

Note also that expected losses as defined here (as rational expectations) are different in general from the reserves for loan losses prescribed by accounting rules for banks. These expected losses are closer to the "statistical provisions" recently introduced by regulators in Spain, as described by Fernández de Lis et al. (2000). Cavallo and Majnoni (2001) discuss loan loss provisions in G10 and non-G10 countries.

For expositional purposes, we first consider the model in the absence of adjustment costs, which is essentially myopic. We then turn to the full dynamic model with adjustment costs.

2.2. Optimization with no adjustments costs

In this section, we model two of the three capital-related costs that banks face: the cost of holding capital and the cost of failure. First, it is costly for the bank to hold capital, and the cost is proportional to the level of capital of an operating bank. This cost is not necessarily the nominal return on capital, but may be the difference between the cost of capital funding and funding through other means such as debt. [8] We express this cost of capital as

$$C_c = \max\{c_c K_t, 0\}. \tag{9}$$

The stochastic component of losses is assumed to have a time-invariant continuous cumulative distribution function $F(u_t)$, which is known at the beginning of the period. [9] Thus, the expected value of this cost at time t is given by

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[8] Jensen (1986), for instance, argues that substituting equity with debt can reduce agency costs of free cash flow.

[9] At least at the aggregate level, there is evidence of cyclical stability in the uncertain component of economic returns. For instance, Price (1994) finds that the process for UK GDP is homoskedastic, and analogous results may be obtained for the United States (see Estrella, 2003). Also, Nickell et al. (2000) successfully model cyclical fluctuations in credit rating transition matrices using a homoskedastic multinomial ordered probit model.

$$E_t C_c = c_c \int_{-\infty}^{K_{t-1}+R_t-E_tL_t} (K_{t-1} + R_t - E_tL_t - u_t)F'(u_t) \, du_t. \tag{10}$$

Second, the bank faces a cost of failure, which is proportional to the absolute value of the negative net worth of a bank that fails. [10] This measure corresponds to the costs of bankruptcy, including loss of charter value, reputational loss, and legal costs. It is represented as

$$C_f = \max\{-c_f K_t, 0\} \tag{11}$$

and its expected value at time t is

$$E_t C_f = c_f \int_{K_{t-1}+R_t-E_tL_t}^{\infty} (-K_{t-1} - R_t + E_tL_t + u_t)F'(u_t) \, du_t. \tag{12}$$

In the absence of adjustment costs, the bank may fully adjust to the desired level of capital in the current period. Thus, the bank's objective is to select a level of external capital raised so as to minimize the expected value of the costs of capital and failure. Assume that the levels of loans and deposits are given, for example, that they are determined as in Bernanke and Blinder (1988). The bank's problem is

$$\min_{\{R_t\}} C = E_t(C_f + C_c). \tag{13}$$

This problem may be solved by substituting (10) and (12) into (13) and solving the first-order conditions.

Proposition 1 [11]
(a) *C is a "U-shaped" convex function of R_t.*
(b) *C has a global minimum at $R_t^* = K^* - K_{t-1} + E_tL_t$, where K^* is defined implicitly by*

$$P(K_t < 0) = P(u_t > K^*) = 1 - F(K^*) = \frac{c_c}{c_f + c_c}. \tag{14}$$

(c) *The optimal expected level of capital is constant*:

$$E_t K_t = K_{t-1} + R_t^* - E_tL_t = K^*. \tag{15}$$

Proposition 1(b) states that the solution to the model with no adjustment costs is equivalent to a var approach to risk in which the probability of failure, which is frequently viewed as a subjective parameter, is determined endogenously. Var is generally defined as the level at which the probability that losses will exceed var is no greater than, say, α. [12] From (14), we have that

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[10] It may be argued that there is also a fixed cost component to bank failures. Some important costs, however, such as reputational costs to managers, would seem to be roughly proportional. Fixed costs are not included in the model for tractability. It may also be argued that the proportional costs of capital and failure would vary over an economic cycle. For instance, the cost of capital may be proportionately higher in an economic downturn. If so, some of the cyclical fluctuations derived later in this paper may be exacerbated. Constant proportions are adopted in the model in order to obtain a closed form solution.

[11] Proofs of the propositions are given in Appendix A.

[12] See, for instance, Jorion (1997, Eqs. (5.3) and (5.4)). Duffie and Singleton (2003, Chapter 2) discuss var and, more generally, the economics of risk management.

$$P(u_t > K^*) = P(L_t > E_t L_t + K^*) = c_c/(c_f + c_c), \tag{16}$$

which implies that if we set $\alpha = c_c/(c_f + c_c)$, then

$$\mathrm{var}_t = E_t L_t + K^*. \tag{17}$$

Thus, a problem in which the objective function includes a desire to minimize a conditional tail expectation corresponding to bank failure is seen to be equivalent to setting var at a specific level, once the cost of capital is taken into account. [13]

Note also that (15) and (17) imply that

$$R_t^* = \mathrm{var}_t - K_{t-1}, \tag{18}$$

which helps illustrate the importance of distinguishing between stocks and flows of capital in the model. The optimal expected level of capital is constant and hence uncorrelated with var, but (18) shows that the optimal flow of external capital varies one-for-one with var, given the previous period's level of capital.

2.3. Capital regulation

Thus far, we have approached capital from the perspective of the rational optimizing bank. In a world of perfect competitive equilibrium, that perspective would be sufficient and there would be no need for capital regulation. In this section, we consider the perspective of the regulator. What are the regulator's preferences and how are they motivated? Under what conditions is regulatory intervention acceptable or advisable? If intervention is required, what tools are available to the regulator?

We assume that the regulator is driven by two goals emanating from aspects of social welfare: to reduce the instability and social costs associated with bank failures and to avoid distortions away from the equilibrium level of output. There is evidence that these goals are in fact important to the regulatory community. Speaking to the first goal, Greenspan (1998) says that "When the [1988 Basel] Accord was being crafted, many supervisors may have had an implicit notion of what they meant by soundness – they probably meant the likelihood of a bank becoming insolvent." This statement is consistent with the emphasis on the probability of failure in the results of Section 2.2.

As to the second goal, Crockett (2000) expresses concern that binding capital requirements may exacerbate the business cycle, particularly at the trough. He argues that "Strengthening the macro-prudential orientation of the regulatory and supervisory framework is important because of the costs and nature of financial instability [i.e., excessive cyclicality]. The main costs take the form of output losses." The implicit positive connection between bank lending and output is consistent with eco-

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[13] A recent paper by Artzner et al. (1999) argues that var is not – by their definition – a coherent measure of risk. They argue in favor of the class of coherent measures of risk, which includes conditional tail expectations. The reader is referred to the mathematically rigorous treatment in Artzner et al. (1999) for details of these distinctions. Note, however, that in the present model, var arises endogenously by minimizing the weighted sum of two conditional tail expectations (Eq. (13)), one involving failure, as in Artzner et al. (1999), and the other involving the cost of capital.

nomic theory and evidence. For example, Bernanke and Blinder (1988) propose a model in which the demand for bank loans depends positively on the level of output. A downward shift in the supply of bank lending leads to reductions in the equilibrium levels of both loans and output.

This section extends the model presented so far to include explicit regulatory preferences based on the above considerations and to highlight the relationship between risk, the supply of loans, and output. The extended model suggests that the probability of bank failure is positively related to the level of lending, that capital regulation may affect the levels of lending and output, and that capital regulation may be used in some circumstances to achieve social goals.

Let

$$\pi(V_t) = P(u_t > K^*) = P\left(\eta_t > \frac{K^*}{V_t}\right) \tag{19}$$

be the probability of failure as a function of the level of loans. The last equality follows from (8). If the random variable η_t is absolutely continuous and its density is positive, then for a given level of expected capital K^*,

$$\pi'(V_t) > 0. \tag{20}$$

Recall that an optimizing bank chooses R_t so that $E_t K_t = K^*$, where $P(u_t > K^*) = c_c/(c_f + c_c)$. If $\alpha^u \equiv c_c/(c_f + c_c)$ and V_t^u represent the unconstrained levels of the probability of failure and lending, respectively, the optimality condition may be expressed as $\pi(V_t^u) = \alpha^u$.

Now suppose the preferences of the regulator are guided by the principles enunciated by Greenspan and Crockett. The regulator sets capital requirements implicitly by imposing a ceiling on the probability of failure, $\pi(V_t) \leqslant \alpha^R$. If the requirement is binding, we assume in this section that the bank reacts by lending less, since raising more capital would increase expected costs (Eq. (13)). [14]

Let $p_t(\alpha^R)$ denote the actual probability of failure associated with a regulatory ceiling α^R. If $\alpha^R \geqslant \alpha^u$, the unconstrained optimum prevails, $p_t = \alpha^u$, and $p_t' = 0$. If $\alpha^R < \alpha^u$, the requirement is binding, $p_t = \alpha^R$, and $p_t' = 1$.

Output may also be affected by the regulatory requirement. As in Bernanke and Blinder (1988), output Y_t is positively related to lending, which is in turn positively related to the likelihood of failure. Thus, $Y_t = Y_t(V_t) = Y_t(\pi^{-1}(p_t(\alpha^R)))$. Let $y_t(\alpha^R) \equiv w(Y_t)$, where w is a monotonically increasing transformation representing the value of output from the point of view of the regulator. As in the case of p_t, the unconstrained level of y_t holds when the regulation is not binding ($\alpha^R \geqslant \alpha^u$), so that $y_t' = 0$. When $\alpha^R < \alpha^u$, we assume that $y_t' > 0$ and $y_t'' < 0$. That is, as the capital requirement is tightened beyond the point at which it becomes binding, the marginal cost of foregone output increases with each further tightening.

Suppose that, in line with Greenspan and Crockett, the regulatory or social objective is to select a level of α^R so as to maximize

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[14] The alternative of raising more capital is examined in Section 3.3.

$$W_t = y_t(\alpha^R) - p_t(\alpha^R). \tag{21}$$

There are two possible cases. First, suppose that $y_t'(\alpha^R) > p_t'(\alpha^R)$ for all $\alpha^R < \alpha^u$, so that the output cost of tightening the regulation is always higher than the benefit. Then the regulator prefers not to impose a binding capital requirement. In contrast, suppose that $y_t'(\alpha^R) < p_t'(\alpha^R)$ for some α^R. Then there is an interior solution to the regulator's problem and the optimal level of regulation α^R is given by the first-order condition is $y_t'(\alpha^R) = p_t'(\alpha^R)$. In general, the regulator prefers not to distort output, unless the relative marginal cost of failure is too high.

Regulation may also be effective when the costs involved in the bank's optimization process are distorted or mismeasured. Suppose that the bank's estimate of the cost of failure is $\hat{c}_f < c_f$, where c_f is the social cost as perceived by the regulator. This situation may occur, for instance, if deposit insurance is mispriced. The bank will then have an incentive to lend at a level consistent with

$$\pi(V_t) = \frac{c_c}{\hat{c}_f + c_c} > \frac{c_c}{c_f + c_c}, \tag{22}$$

implying that $V_t > V_t^u$. The higher level of lending is associated with a probability of failure that is inappropriately high from the point of view of the regulator. In this case, the regulator could impose a requirement of the form $\pi(V_t) \leqslant c_c/(c_f + c_c)$, which would restore the socially optimal probability of failure and the corresponding level of lending.

If the regulator decides to impose a regulatory capital requirement, the requirement may be expressed in various equivalent ways. As already indicated, one form that is directly in line with the remarks by Greenspan (1998) is

$$\pi(V_t) \leqslant \alpha^R, \tag{23}$$

which sets an upper bound on the probability of failure. An alternative equivalent form employs the concepts of initial capital I_t and var, as defined earlier. Specifically, Eq. (15) may be written as $I_t = K_{t-1} + R_t^* = E_t L_t + K^* = \text{var}_t$, so that $\pi(V_t) \leqslant c_c/(c_f + c_c)$ may be expressed as

$$I_t \geqslant \text{var}_t. \tag{24}$$

It is thus understandable that var constitutes an attractive benchmark for regulatory design, at least in the static framework examined so far. [15]

Another alternative formulation may be based, at least in principle, on the accounting concept of loan loss reserves (LLR). If accounting rules require that loan loss reserves be consistent with expected losses, as defined here, the "unexpected loss" component of var (or UL, see Basel Committee, 1999) coincides with optimal expected capital in the case with no adjustment costs. More precisely, if $\text{LLR}_t = E_t L_t$, then $\text{UL}_t = \text{var}_t - \text{LLR}_t = \text{var}_t - E_t L_t = K^* = E_t K_t$. Thus, (24) may be expressed as

$$E_t K_t \geqslant \text{UL}_t, \tag{25}$$

which is more in line with the framework of the Basel Committee (1999).

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[15] If deposits are fixed, a requirement on I is also equivalent to a requirement on the "leverage ratio" $I/(I + D)$. Note, however, that the minimum requirement is not constant, but is a function of var.

2.4. Optimization with adjustment costs

We now bring adjustment costs explicitly into the model. These costs apply both when the firm is raising new external capital and when it is shedding external capital. Why do these entry and exit costs arise?

Several reasons have been given for entry costs in the corporate finance literature. For instance, these costs may be associated with asymmetric information. Equity is a form of capital for which monitoring costs are high, and the firm has an informational advantage over public investors as to the value of its own equity (Myers and Majluf, 1984). A related reason is that the issuance of equity may send a signal to the market that it is being done at time most advantageous for the company and not necessarily for outside investors (Winter, 1994). A specific example of this type of signal occurs when a bank is attempting to replenish capital after suffering severe losses. A third reason is the "trapped equity effect" of dividend taxation (Winter, 1994). Once equity is raised, it is costly for investors to obtain returns in the form of dividends, which are subject to high taxes as compared with other forms of income.

Exit costs may arise for various reasons as well. For instance, the firm may be reluctant to shed equity if there is a good chance that it may have to bear round trip costs of raising the equity again in the short term (Winter, 1994). A second reason is the so-called stock repurchase premium (McNally, 1999). If the company opts for shedding equity through a stock repurchase, the market may interpret this as a signal that the company's stock is undervalued. The stock price may rise temporarily for non-fundamental reasons and the cost of the buyback may increase. Finally, an important cost of shedding equity comes from pressure from regulators, supervisors and market participants to maintain clearly sound levels of capital.

For all of the foregoing reasons, we model adjustment costs in the following simple form:

$$C_a = \tfrac{1}{2} c_a R_t^2. \tag{26}$$

Of course, there is no assurance that the costs will be symmetrical, as assumed in (26). For instance, one might suspect that the costs of raising a given amount of new capital are larger than the costs of shedding the same amount. Nevertheless, since the objective is to study cyclical patterns and not to measure these costs precisely, (26) seems like a reasonable approximation that preserves the qualitative behavior while allowing for an explicit solution to the general model. Asymmetrical adjustment costs would tend to skew cyclical fluctuations in one direction or the other, but would not eliminate the existence of a cycle or fundamentally change its length.

In fact, we adopt a similar approximation to the cost function C of the previous section before constructing the full model. The reason is that we would like to have linear-quadratic form for which the optimal solution may be computed explicitly. Thus, we use a second-order Taylor approximation to the U-shaped function C around the optimum value of R in the case with no adjustment costs:

$$C \approx \tfrac{1}{2}(c_f + c_c)F'(K^*)(K_{t-1} + R_t - E_t L_t - K^*)^2. \tag{27}$$

Note that the constant term is irrelevant for optimization and that the first-order term disappears because the approximation is taken around the optimum.

Combining these approximations in an infinite-horizon objective function with time-discount factor β, the bank's intertemporal problem becomes

$$\min_{\{R_{t+i}\}} E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2}(K_{t+i-1} + R_{t+i} - L_{t+i} - K^*)^2 + \frac{a}{2}R_{t+i}^2 \right\}, \tag{28}$$

subject to (4), where $a = c_a/[(c_f + c_c)F'(K^*)]$.

One strategy for solving this optimization problem is to use Eq. (4) to solve for R_t and substitute the result in (28). Taking derivatives with respect to K_{t+i}, $i = 0, 1, \ldots, \infty$, we obtain the first-order conditions (or Euler equations)

$$E_t \left\{ K_{t+i+1} - \frac{\gamma}{\beta}K_{t+i} + \frac{1}{\beta}K_{t+i-1} - \frac{1}{\beta}L_{t+i} + L_{t+i+1} + \frac{K^*}{\beta a} \right\} = 0, \tag{29}$$

$i = 0, 1, \ldots, \infty$, where $\gamma \equiv \frac{1}{a} + 1 + \beta$. These allow for the solution of K in terms of the L.

First note that there are two solutions to the characteristic polynomial $\lambda^2 - \frac{\gamma}{\beta}\lambda + \frac{1}{\beta} = 0$ corresponding to K in (29), which satisfy $0 \leqslant \lambda_1 < 1$ and $\lambda_2 = \frac{1}{\beta\lambda_1} > 1$. As is customary in rational expectations models, the first root it solved backward and the second root is solved forward in terms of K and L. In addition, the second root may be expressed in terms of the first, so that only the first is needed to write the solution. [16] The optimal paths of the level of capital and net new external capital are given in the following result.

Proposition 2. *The solution to the optimization problem in expression* (28), *subject to* (4), *satisfies*

$$K_t = (1 - \lambda_1)K^* + \lambda_1 \left\{ K_{t-1} + E_t \sum_{i=0}^{\infty} (\lambda_1 \beta)^i (\beta L_{t+i+1} - L_{t+i}) \right\} - u_t \tag{30}$$

and

$$R_t = (1 - \lambda_1)(K^* - K_{t-1}) + E_t \sum_{i=0}^{\infty} (1 - \lambda_1)\lambda_1^i \beta^i L_{t+i}. \tag{31}$$

Note also that the root λ_1 is a function of the adjustment cost parameter a such that $a = 0 \Rightarrow \lambda_1 = 0$ and $a \to \infty \Rightarrow \lambda_1 \to 1$.

Eqs. (30) and (31) contain an infinity of expectational terms. [17] Until the expectations are defined, these equations do not constitute observable reduced form relationships between losses and the capital levels and flows. Nevertheless, the

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[16] See Appendix A for an exact expression for λ_1.

[17] The infinite horizons of the objective function and of the solution are consistent with a view that failed banks do not disappear, but are taken over by other institutions.

expressions may be used to gain some intuition into the rational expectations solution.

For instance, recall that when there are no adjustment costs, the expected level of capital every period is set to K^*, and new capital raised is whatever it takes to bring expected capital to that desired level. Note that (30), apart from the disturbance term, is a weighted average of K^* and of a term involving last period's capital and the present values of expected future losses. These expected losses enter as present values of generalized first differences of losses. When $a = \lambda_1 = 0$, the first term receives the full weight. As adjustment costs increase, the second term receives greater weight and the optimal level tends to converge to K^* only with a lag. In general, optimum expected capital will differ from K^* ($= \mathrm{UL}$) and, when losses turn out to be larger than expected, requirement (25) and its equivalent form (24) in terms of var will tend to be violated.

The first term of Eq. (31) conveys the partial adjustment character of the solution. [18] The second term is a long-run weighted average of discounted expected losses with weights $(1 - \lambda_1)\lambda_1^i$, which add up to 1. When $a = 0$, all the weight is on contemporaneous expected losses and the equation reduces to $K^* - K_{t-1} + E_t L_t$, the static optimum. As a goes to infinity, the weights on the individual discounted losses become more uniform and also much smaller. The second term gains in importance as a increases from zero, but as a approaches infinity, the cost of raising external capital becomes prohibitive, and both terms go to zero.

Note that with adjustment costs, the myopic regulatory objective function (21) has to be modified to be forward-looking. A natural extension is to retain (21), but to define $y_t(\alpha^R) = w\left(\sum_{i=0}^{\infty} \beta^i Y_{t+i}(V_{t+i})\right)$ and $p_t(\alpha^R) = \max\{\pi(V_{t+i})\}_{i=0}^{\infty}$, both under the assumption that $\pi(V_{t+i}) \leqslant \alpha^R$ in each period. Thus, the regulator focuses on the present value of current and future output and on the worst-case probability of failure in the foreseeable future. With these definitions, determination of the regulatory optimum α^R follows essentially the same principles as in the case with no adjustment costs.

3. Cyclical behavior

To model cyclical behavior, we need a specification in which losses tend to behave cyclically in a way that is at least partly predictable. [19] We then insert this process into the optimal solutions given by (30) and (31), and examine the consequences for K_t, R_t and ΔK_t over time.

Thus, let the process for losses be represented by

$$L_t = (2\cos\omega)L_{t-1} - L_{t-2} + u_t \tag{32}$$

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[18] In the context of an empirical model, Hovakimian et al. (2001) use a partial adjustment approach to the debt–equity choice problem.

[19] Statistical analysis has shown that the business cycle is at least partly predictable. See, for instance, Stock and Watson (1989, 1993), Estrella and Mishkin (1998), and Estrella (2003).

with $0 < \omega < \pi$, where u_t is white noise. Expected losses then follow a cycle of frequency ω with constant amplitude. It is normally desirable that the roots of a process like (32) lie outside the unit circle, which implies that the process is stationary. For present purposes, it is more convenient to use the limiting case in which the roots lie on the unit circle so that cycles remain undamped and their properties are clearer to examine. Section 3.3 considers the stationary case.

The assumption of a sinusoidal pattern is not entirely realistic, but is used to gain insights into optimal behavior under truly cyclical conditions. Moreover, the assumption is justified by the fact that any stationary time series may be expressed as a linear combination of sinusoids (see, for example, Jenkins and Watts, 1968). Using multiple sinusoids could lead to more realistic patterns, but would also complicate the analysis and obscure the intuition behind the results that follow.

In addition to having convenient cyclical properties, expected future losses under this process have a simple form,

$$E_t L_{t+n} = \frac{m_2^{n+2} - m_1^{n+2}}{m_2 - m_1} L_{t-1} - \frac{m_1 m_2^{n+2} - m_2 m_1^{n+2}}{m_2 - m_1} L_{t-2}, \tag{33}$$

where m_1 and m_2 are $\exp(\pm i\omega)$, which correspond to the complex conjugate roots of Eq. (32). This expression makes it possible to transform (30) and (31) into observable equations. For simplicity, we assume that the unconditional mean of L is zero. Since L represents the negative of net income, its unconditional mean is likely to be negative. However, results with non-zero unconditional mean are qualitatively very similar. [20]

3.1. Optimal behavior with cyclical losses

Suppose that losses are cyclical in the manner defined in Eq. (32). Since expectations of losses at all horizons are given by Eq. (33), we replace the expectation terms in expressions (30) and (31) to obtain equivalent expressions based only on observable variables. In particular, we have:

Proposition 3. *Suppose that losses follow the cyclical pattern* (32), *where the shock u_t has a constant distribution F. Then the optimal level of capital and the optimal net flow of external capital are given by*

$$K_t = (1 - \lambda_1)K^* + \lambda_1 K_{t-1} - E_t L_t + \delta_1 L_{t-1} + \delta_2 L_{t-2} - u_t \tag{34}$$

and

$$R_t = (1 - \lambda_1)(K^* - K_{t-1}) + \delta_1 L_{t-1} + \delta_2 L_{t-2}, \tag{35}$$

respectively, where $\delta_i = \delta_i(a, \beta, \omega)$ (with $\delta_1 = 2\cos\omega$ and $\delta_2 = -1$ when $\lambda_1 = 0$ and $\delta_1 = \delta_2 = 0$ when $\lambda_1 = 1$). Exact expressions for the δ_i are given in Appendix A.

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[20] For a detailed discussion of the implications of a non-zero unconditional mean for expectations of the variables in the model, see Estrella (2003).

The results of Proposition 3 may be used to construct numerical illustrations and also to examine in more detail the cyclical relationships among the variables. We start with the illustrations.

Fig. 1 shows the effects of a stochastic shock to L_t of size $\sin \omega$. A shock of this magnitude produces a cycle in losses of amplitude 2 (losses are in the range of ± 1), which is a convenient benchmark. The choice of $a = 5$ is somewhat arbitrary, but is roughly consistent with empirical estimates presented later in Section 4.

The figure also assumes that the frequency ω is $2\pi/20$ and that the discount factor β has a value of $1/1.01$. These assumptions correspond to a quarterly model in which the cycle lasts for 5 years (20 quarters) and the annual discount rate is about 4%. The starting values for all variables in Fig. 1 are their respective unconditional means.

Panel A of Fig. 1 shows the pattern of the time series for losses, following the initial shock to that series, as well as the reaction of the optimizing bank in terms of external capital raised (R). The optimal amount of external capital raised rises

Cycle length is 20 quarters, relative adjustment cost is 5.00

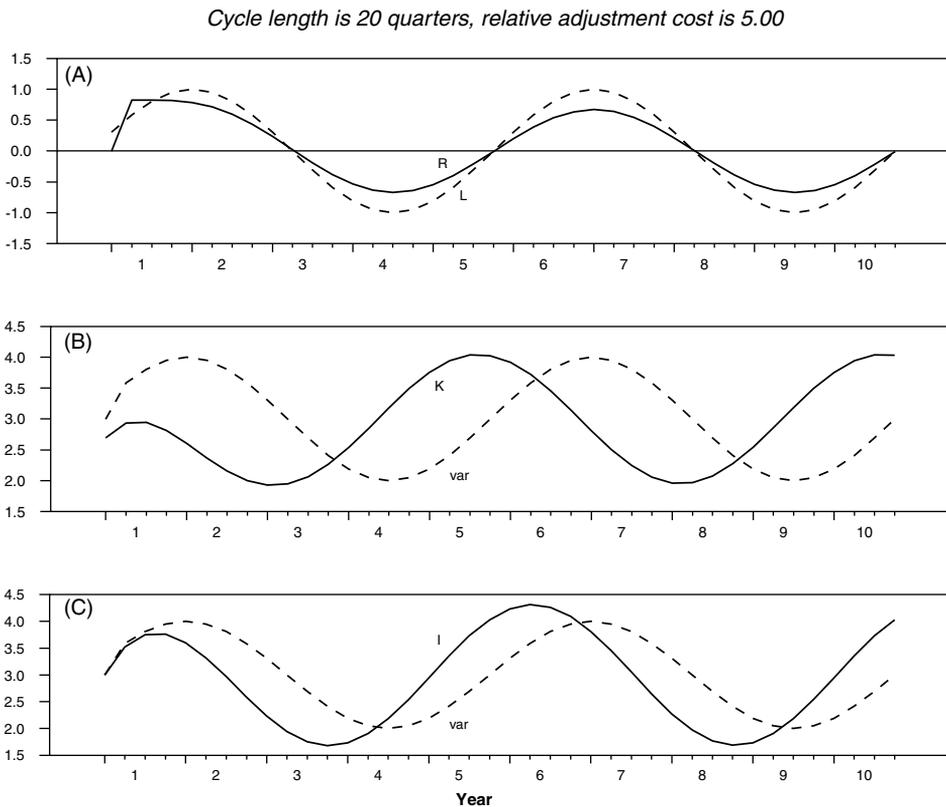Fig. 1. Responses to a stochastic shock to L_t quarterly data, initial shock of size $\sin \omega$ to L_t in quarter number 1: (A) losses and external capital raised, (B) capital level and var, (C) initial capital and var.

sharply in the second quarter in response to the shock to losses one period earlier. Within a few quarters, the capital raised falls into a cyclical pattern whose timing is very close to that of losses, but with smaller amplitude.

Panel B compares var with the overall level of capital. We have seen that the pattern of var is essentially the same as that of losses, except that var is transposed upward by a constant amount K^*, which we assume has a value of 3 for illustrative purposes. [21] The level of capital moves in a cycle that is out of phase with the cycle of losses. We will see shortly that the lag in the cyclical movements of the level of capital, relative to losses, bears a consistent relationship to the length of the cycle. This lag is easy to see with the parameter values in Fig. 1, but it holds in general. In addition, the level of capital sometimes moves in the same direction as var, and sometimes in the opposite direction. On balance, we will see that this relationship is negative.

In Panel C of Fig. 1, we see a comparison between initial capital and var, which corresponds to the capital requirement (24) derived from static optimization. Abstracting from adjustment costs, it would be reasonable to require the bank to hold initial capital at least in the amount of var. With adjustment costs, however, it is optimal for the bank to adjust its capital with a lag, as seen in Panel C.

The lag results in a conflict between var and optimal initial capital. The shock to losses raises the value of var more rapidly than the optimal level of initial capital (which also rises). The value of var continues to exceed the optimal level of capital over approximately one half of the cycle, with the opposite relationship holding over the other half. This result suggests that a minimum capital requirement based on var would be binding following contractionary shocks, in which losses are higher than anticipated, and loose after expansionary shocks, when losses fall short of expectations.

From the point of view of the regulator, var is neither more nor less conservative than optimal capital on a consistent basis. A var-based requirement is consistent with assuming away the effects of adjustment costs. However, the degree to which a var requirement is binding depends on whether the initial shock to losses is positive or negative, and on the current stage within the resulting cycle.

Some intuition for the foregoing results may be obtained by considering two opposing influences on capital. If the economy experiences a negative shock, bank losses tend to be higher, which tends to deplete capital. However, measures of risk tend to indicate a greater need for capital and the natural reaction is for the bank to raise capital. In the absence of adjustment costs, the bank fully offsets losses and the level of capital remains constant. However, if there are adjustment costs, the reaction is somewhat delayed and it is optimal to allow capital to remain below the unconditional mean level for a period, as shown in Panel B.

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[21] This value is consistent with an infinity of parametric assumptions and distributions. It is obtained, for example, by assuming that the innovation u_t follows a normal distribution with standard deviation 1.46, and that $c_c/(c_f + c_c) = 0.02$.

With adjustment costs, capital tends to vary in a pattern that is not exactly synchronized with var. One result is that there tends to be a conflict between the behavior of an essentially myopic measure of risk, like var, and the behavior of optimal bank capital over the cycle.

3.2. Covariances and lags between optimal capital and var

The important questions regarding the procyclicality of var-based requirements focus on the relationship between optimal capital and var at cyclical frequencies. Since we have assumed that our cycle has frequency ω, this is the relevant frequency for the analysis of covariation. This section uses frequency domain techniques to examine the relevant relationships for in-phase components of frequency ω and to compute the exact phase lag between optimal capital and var, which was observed in Fig. 1.

First, we compute the equivalent of regression coefficients for each of the key optimal variables with respect to var. These coefficients are obtained from the lag structures of Eqs. (34) and (35). Thus, let Y represent one of R, ΔK, K or I, and let

$$Y_t = h_{YX}(L)X_t. \tag{36}$$

Note that Eqs. (34) and (35) are of this form with $Y = K$ and R, alternatively, and $X = L$. Note also that a constant may be added to the right-hand side of Eq. (36) without fundamentally affecting the results. The coefficient of a regression of the component of Y of frequency ω on the in-phase component of X of frequency ω is given by

$$b_{YX}(\omega) = \text{Re}(h_{YX}(e^{-i\omega})), \tag{37}$$

where "Re" indicates the real part of the complex number in parentheses. [22]

Proposition 4. *In spectral regressions of the sort described above,*

(a) *the coefficient of R regressed on L is non-negative for any frequency ω,*

$$0 \leqslant b_{RL}(\omega) \leqslant 1,$$

(b) *the coefficient of ΔK regressed on L is non-positive for any frequency ω,*

$$-1 \leqslant b_{\Delta KL}(\omega) \leqslant 0,$$

(c) *the coefficient of K regressed on L is non-positive for any frequency ω,*

$$b_{KL}(\omega) \leqslant 0, \quad and$$

(d) *the coefficient of I regressed on L is less than or equal to unity for any frequency ω,*

$$b_{IL}(\omega) \leqslant 1.$$

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[22] See, e.g., Jenkins and Watts (1968, Section 8.3.1).

Expressions for the coefficients are found in Appendix A. If the regressor in Proposition 4 were var instead of L, the regression coefficients would have the same signs, but would be closer to zero. To see this, note that Eq. (17) implies that

$$L_t = \text{var}_t - K^* + u_t. \tag{38}$$

Thus, L and var differ by a constant plus an innovation that is uncorrelated with var, and we have effectively an errors-in-variables problem in which the regression coefficient is biased toward zero.

The signs of the relationships between R and either L or var are easily verified in Fig. 1 (Panels A and B). The signs of the coefficients of K and I are harder to visualize in Panels B and C because, although the patterns are similar to those of L and var, they appear to be transposed in time by a few periods. We can use the frequency domain specification to examine the lengths of these lags.

The lag in the relationship between K and L may be calculated explicitly from the phase function, which is defined as

$$\phi_{KL}(\omega) = \arg(h_{KL}(e^{-i\omega})). \tag{39}$$

Here, "arg" is the argument of the complex number in parentheses, that is, the angle that appears in the exponent of the polar form of the number. The ratio

$$-\phi_{KL}(\omega)/\omega \tag{40}$$

is a measure of the phase lag between K and L, measured in periods. [23] Thus, if a cyclical peak in L_t occurs at time t_0, then a cyclical peak in K_t occurs at time $t_0 + (-\phi_{KL}(\omega)/\omega)$.

Proposition 5
(a) *When $a > 0$, the phase lag between K and L lies in the interval*

$$\frac{1}{\omega} \tan^{-1}\left[\frac{\beta \sin \omega}{1 - \beta \cos \omega}\right] \leqslant -\phi_{KL}(\omega)/\omega \leqslant \frac{1}{\omega} \tan^{-1}\left[\frac{\sin \omega}{1 - \cos \omega}\right]. \tag{41}$$

(b) *Since $\beta \approx 1$,*

$$-\phi_{KL}(\omega)/\omega \approx \frac{1}{\omega} \tan^{-1}\left[\frac{\sin \omega}{1 - \cos \omega}\right] = -\frac{1}{2} - \frac{q}{4}, \tag{42}$$

where $q = 2\pi/\omega$ is the length of the cycle of frequency ω, in periods.

In Fig. 1, $q = 20$ and the phase lag for K in Panel B is about -5.5 quarters (periods). This means that K_t peaks about 5.5 quarters before L_t and has a trough one-half cycle later, or about 4.5 quarters after L_t peaks.

A look at Panel C suggests that initial capital I, like K, leads losses L_t over the cycle. This relationship holds generally, but in contrast to the phase lag for K, the

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[23] Sargent (1979, Section XI.6) provides a helpful discussion of phase lags.

exact phase lag for I depends more substantively on the adjustment cost parameter a. As a approaches zero, the model approaches the static case, in which optimal initial capital equals var. In that extreme case, the phase lag is zero. In the case illustrated in Fig. 1, with $a = 5$ and $q = 20$, the phase lag for initial capital is -2.9, so that I peaks about three quarters before var.

3.3. Constrained optimization with a var requirement

In Section 2.3, the optimizing bank responds to a binding capital requirement by reducing the supply of lending, which has a negative effect on economic output. In this section, we examine the additional costs incurred by an optimizing bank if it maintains the unconstrained supply of lending but is required by the regulator to set initial capital to be at least var. The relative magnitude of the additional costs incurred suggests that the net effect of this type of cyclically binding regulation is likely to be a reduction in lending and output, as assumed earlier.

As before, the bank minimizes the objective function (28) subject to the dynamic identity (4), but now constraint (24) requiring that initial capital be at least var is also imposed. An analytical solution is not available under this additional constraint, but the problem may be solved numerically, particularly if the cycle for losses is mean-reverting. Thus, in contrast to the cyclical model of losses employed so far in Section 3, we now assume that the AR(2) process for losses has roots outside the unit circle. In particular, we take the roots to be $\frac{4}{3} \exp(\pm i\omega)$ so that m_1 and m_2 are $\frac{3}{4} \exp(\pm i\omega)$. With these values, the variables in the model essentially return to their equilibrium levels within a 40-period horizon following a shock, allowing for straightforward numerical solution.

Fig. 2 shows the behavior of var and of unconstrained and constrained optimal initial capital when we take $a = 5$ and $\omega = 2\pi/20$, as so far in this section. Note first that when the cycle is dampened, the conflict between optimal initial capital and var is dominated by the first part of the cycle. When the shock is adverse, as in the figure, the constraint tends to be binding over most of the period of adjustment back to equilibrium.

The var constraint is binding in various ways. In a direct sense, we see that var exceeds the unconstrained optimum for 13 quarters after the initial shock. Second, the constraint is binding in the sense that constrained capital is higher than the un-constrained level throughout the whole period. Third, constrained capital is strictly higher than var starting with the ninth period after the shock, even though var is at or below its equilibrium level.

The severity of the constraint is reflected in the values of the objective function corresponding to the three paths in Fig. 2. The absolute levels of the function are not meaningful, but the relative levels for the three paths provide some indication of the cost of moving from optimum to var and of the improvement from optimizing subject to the inequality constraint based on var. The unconstrained optimum level is 1.079, var is 1.844, and the constrained optimum is 1.842. Most of the slight improvement from var to the constrained optimum (88%) is attributable to reduced adjustment costs.
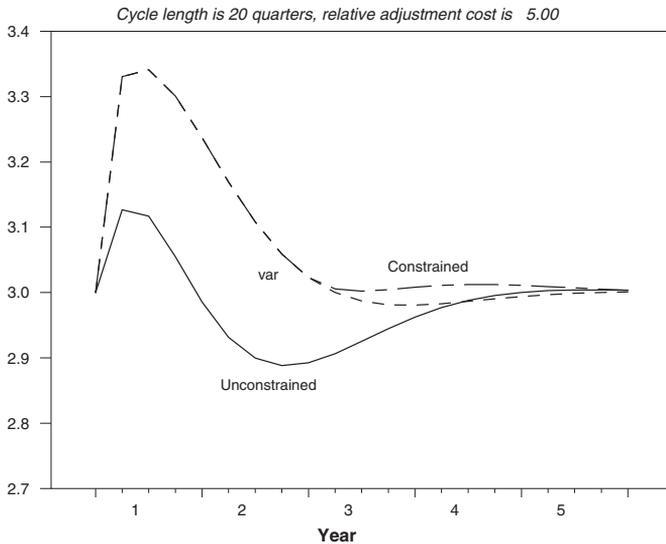
Fig. 2. Optimum initial capital: unconstrained, var, constrained quarterly data, initial shock of size $\sin \omega$ to L_t in quarter number 1.

3.4. Discussion of results

The conflict between optimal capital and a var-based minimum requirement that we see in Fig. 1 (and in Propositions 4(c) and (d)) may manifest itself in practice in two forms, depending on whether the economy is expanding or contracting.

In the contraction phase of the cycle, bank capital falls below the var level and the potential is for a credit crunch. If the bank were to maintain its level of lending, we have seen in Section 3.3 that costs would be prohibitively high. However, if it chooses to hold down its costs, as in Section 2.3, it is driven to cut back on risky assets, such as commercial loans.

The potential for this type conflict may be reduced by judicious calibration of the var-based minimum requirement. It is important to consider not only the unconditional average relationship between var and the level of capital, but also the relationship conditional on an economic downturn.

During the expansion phase of the cycle, the level of optimal capital may greatly exceed a var-based minimum requirement, which creates a potential moral hazard problem. The level of optimal capital may be so far above the var minimum that the bank may be tempted to let capital fall toward the minimum, or it may use the ''excess capital'' to fund substandard loans. Pillar 2 (supervisory review process) of the new Basel Accord (Basel Committee, 2001c) contains important provisions that could be very useful in dealing with this type of moral hazard problem.

Specifically, Principle 3 states that "Supervisors should expect banks to operate above the minimum regulatory capital ratios and should have the ability to require banks to hold capital in excess of the minimum." The motivation that the Basel

Committee gives for this principle is close to the issues raised in the present paper. For instance, the Committee refers to adjustment costs ("It may be costly for a bank to raise additional capital, especially if this needs to be done quickly or at a time when market conditions are unfavorable.") and to macroeconomic cycles ("There may be risks ... to an economy at large ... that are not taken into account in Pillar 1 [quantitative requirements].").

4. Empirical evidence

In this section, we consider the empirical plausibility of the theoretical model presented in this paper. Ideally, one would estimate the model directly using data for individual banks. However, that strategy is unavailable because of a lack of appropriate data. Accounting data on banks' net income is not reflective of the true stochastic distribution of losses, which plays a central role in the model. The level of income may be accurately represented on average, but the volatility of income and the likelihood of a large loss tend to be greatly understated. Therefore, this section focuses on the empirical verification of a few empirical implications of the model.

In particular, we look at aggregate time series data over a period containing at least one business cycle. Detailed data on changes in capital for all FDIC-insured US commercial banks are available at a quarterly frequency from 1984 to 2001 (from the FFIEC call reports). We use annualized data here to avoid seasonal effects, but quarterly results are similar.

The empirical variables are defined as follows. Net losses, L_t, correspond to the negative of net income, net new external capital raised, R_t, corresponds to net new external capital minus dividends paid, and the level of capital, K_t, corresponds to total equity capital. Two elements of capital are excluded from the analysis, namely changes incident to business combinations, to avoid double counting, and unrealized gains from available-for-sale securities, since we wish to focus on medium-frequency cyclical fluctuations rather than short-term volatility.

Consider the implications of two features of the model. First, the model assumes that adjustment costs are associated with changes in external capital in either direction. Thus, we would expect that net external capital should be close to zero unless net income or losses are very large. If net income is large, there may be a tendency to shed external capital, which is replaced with cost-effective internally generated capital. If losses are very large, there may be a need to raise external capital to return to prudent levels of capitalization. Second, we expect from Proposition 4 that net new external capital raised (R_t) should move in the opposite direction from net income ($-L_t$), while the net change in capital (ΔK_t) should move with net income.

Fig. 3 shows that the values of these three variables over the sample period tend to confirm the regularities described in the previous paragraph. As expected, net external capital raised is relatively close to zero until 1992. After that observation, the economy entered a fairly vigorous economic expansion and net income grew year after year. The banks proceeded to shed some external capital, mainly by expanding their dividend payments. The figure also suggests that net external capital and the
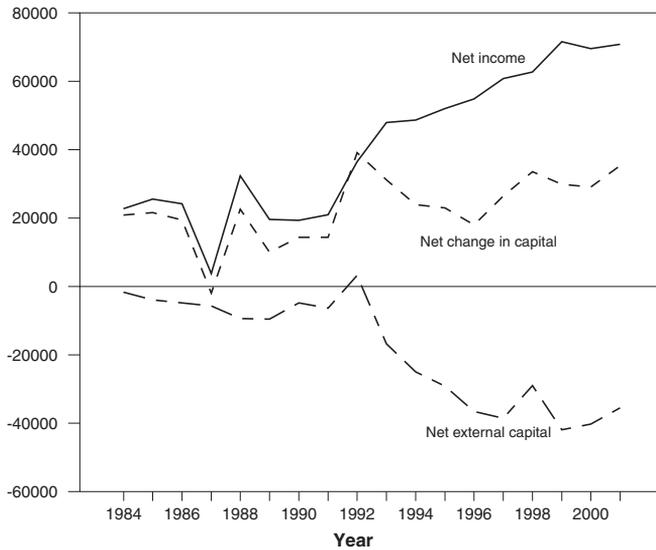
Fig. 3. Bank income and capital flows, all FDIC-insured banks, converted to millions of 1999 dollars using GDP deflator.

net change in capital exhibit the expected relationships to net income. We return to this question shortly.

A few exceptional spikes are noteworthy in Fig. 3. For example, spikes occur in both the net income and net change in capital series in 1987. In that year, several internationally-active banks made substantial provisions for loans to "less developed countries" (LDCs), which had been deteriorating since the early 1980s. The net external capital series shows an upward spike in 1992, most likely as a result of the 1988 Basel Accord and the 1991 US banking legislation, both of which came fully into effect in that year. Other than at these junctures, banks seem to have behaved generally as suggested by the theoretical model for optimal capital.

We now turn to evidence based on the relationships between R_t and L_t, and between ΔK_t and L_t, in the cyclical model of Section 3. The mathematical relationship between the contemporaneous levels of R_t and L_t in Eq. (35) does not appear to be simple, but the spectral linear regression discussed in Proposition 4(a) seems to capture most of the observed relationship between R_t and L_t. The same is true of the regression of ΔK_t on L_t. We see evidence of this in the empirical results of Table 2.

The first three numerical columns of Table 2 show the theoretical values of the spectral regression coefficients $b_{RL}(\omega)$ and $b_{\Delta KL}(\omega)$, calculated from the formulas for the coefficients given in Appendix A. The values of the parameters in the column labeled $a = 5$ are the same as in the earlier numerical illustrations. Under those assumptions, the relationship between R_t and L_t is linear, with a slope of 0.672. That is, for every additional dollar of net income (losses less by $1), the amount of net external capital raised falls by 67 cents.

Table 2
Theoretical and numerical coefficients of regressions of capital flows on L_t

Parameter	Theoretical model			Empirical model
	$a = 1$	$a = 5$	$a = 10$	
Numerical b_{RL}	0.911	0.672	0.506	0.661
Std. error	–	–	–	0.081
R^2	–	–	–	0.806
Numerical $b_{\Delta KL}$	−0.089	−0.328	−0.494	−0.352
Std. error	–	–	–	0.078
R^2	–	–	–	0.559

Note: In the theoretical model, $\beta = 1/1.01$ and $\omega = 2\pi/20$.

For comparison, Table 2 also provides the value of the $b_{RL}(\omega)$ coefficient when a is either 1 or 10. When adjustment costs are lower, the reaction to a given change in income is larger (91 cents per dollar), whereas for larger adjustment costs, the reaction is more sluggish (51 cents per dollar).

The lower panel of Table 2 provides the same type of analysis for the coefficient of the net change in capital, $b_{\Delta KL}(\omega)$. For example, the effect of an additional dollar of net income is to raise capital by 33 cents. The additional dollar flows into capital, but is offset by a 67 cent reduction in new capital raised, as noted above.

The final column in Table 2 presents time domain estimates of the corresponding regression coefficients, using the data described above for US banks from 1984 to 2001. The fit of the equations is relatively good, with an R^2 of 81% for R and L and a significantly positive coefficient estimate, as the theory implies. The empirical estimate of $b_{\Delta KL}$ is significantly negative, in agreement with the implications of the model, and the R^2 is 56%.

The empirical estimates are very similar to those obtained from the model with $a = 5$. In fact, we can use the empirical regression estimate of b_{RL}, together with the expressions for $b_{RL}(\omega)$ and λ_1 in Appendix A, to solve for empirical estimates of λ_1 and a, given β and ω. [24] The estimates, followed in parentheses by standard errors computed by the delta method, are $\lambda_1 = 0.651$ (0.050) and $a = 5.25$ (1.90). The t statistic for a is 2.76, suggesting that adjustment costs play a significant role.

Fig. 4 confirms visually that the linear relationships in the empirical regressions are fairly accurate approximations of the data for banks from 1984 to 2001. The figure shows a scatter plot of the data for both external capital raised and net change in capital, plotted against net income. Also shown in the figure are the fitted values from the empirical regressions in Table 2. As in Fig. 3, the only large outliers correspond to the large loss provisions in 1987, which led to very low net income, and to the effects in 1992 of the 1988 Basel Accord and of the 1991 banking legislation.

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[24] It is customary in empirical estimates of these types of models to take the value of β as given because it can only be estimated very imprecisely and the other estimates are not very dependent on its particular value. See West (1995).
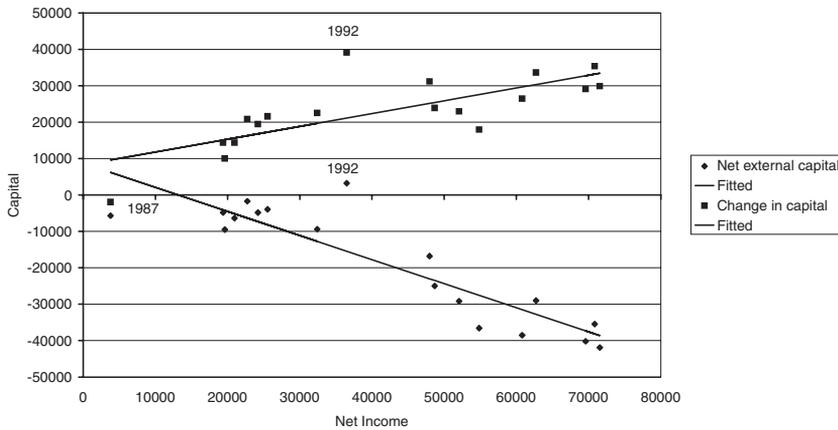
Fig. 4. Net income, net external capital raised, and net change in capital, all FDIC-insured banks, converted to millions of 1999 dollars using GDP deflator.

5. Policy implications

This paper investigates the issue of whether var-based minimum capital requirements are procyclical by comparing the dynamic path of optimum bank capital with the path of var. In the absence of adjustment costs, it is possible to rely on var to define a capital requirement that corresponds to optimal behavior. Specifically, the probability of the bank's failure if the requirement is met will be no greater than the probability associated with the optimum level of capital. In order to use var as a benchmark, however, it has to be compared with initial capital: the bank's level of capital at the beginning of the period after external capital has been raised. Other definitions of capital would produce conflicts with var over the cycle.

However, the empirical evidence suggests that adjustment costs play a significant role in practice. In a dynamic context with adjustment costs, conflicts between optimal capital and var tend to develop if var is used to formulate a capital requirement, even if the concept of initial capital is used in the definition. The problem is that var is an essentially static concept, which does not incorporate information about the costs of adjusting to an optimal level.

The optimum level of capital with adjustment costs tends to lead var by about one quarter of a cycle. If the average level of optimum capital differs from the average var by an amount comparable to one-half the amplitude of either variable, or less, periodic conflicts between the two are likely. Contemporaneous movements in these two variables are sometimes positively, sometimes negatively related, but on average they are negatively related over the cycle.

If var is used as a benchmark for initial capital, as the static model suggests, persistent conflicts between var and optimal initial capital may develop over the cycle. In a realistic framework in which the process for losses is stationary, these conflicts

tend to predominate when the economy and the bank are subject to adverse economic shocks.

Like the optimum level of capital, the change in the optimum level is negatively related to var, but in this case the relationship is clearer and contemporaneous. In contrast, a positive relationship exists between the optimum flow of net new external capital raised and var. These and the foregoing regularities suggest that a var-based minimum requirement is likely to be procyclical if it is applied to the level of capital, but that procyclicality might be avoided if the requirement is applied to capital flows.

The analysis points to various possible solutions to the problem of procyclicality. First, the danger of causing a credit crunch in an economic downturn may be reduced by judicious calibration of a var-based minimum requirement. In the calibration process, the key is to focus on the relationship between var and optimum capital during an economic downturn, rather than simply looking at the unconditional average relationship between these variables.

Second, supervisory review may be very helpful in dealing with the moral hazard problem that confronts banks as the gap between var and optimum capital increases in an economic expansion. At times, the gap may be so large that there is a temptation to follow a shortsighted approach and to let capital fall toward the var. In the model, the fact that the bank is a rational optimizer is axiomatic. However, if a fraction of the banking sector consisted of non-optimizers, there would be scope for the introduction of an agent – a supervisor – to ensure that those banks could be identified. Supervisory review could be used to insure that each bank maintained an adequate buffer between minimum and actual capital even when a formal requirement is non-binding. This type of supervisory strategy is employed in Pillar 2 of the new Basel Accord.

Third, var may be the basis for an acyclical capital requirement – one that does not conflict with optimum levels over the cycle – if it is applied to net external capital raised. In a scheme of this sort, the minimum capital raised could be some fraction of the amount that would be raised in the absence of adjustment costs. In that case, minimum requirements would not tend to conflict with optimum capital at cyclical frequencies, and would be less likely to exacerbate normal cyclical fluctuations.

Appendix A. Proofs of propositions

Proof of Proposition 1. From the definitions, we can compute that

$$C = (c_f + c_c) \int_{K_{t-1}+R_t-E_tL_t}^{\infty} (-K_{t-1} - R_t + E_tL_t + u_t)F'(u_t)\,\mathrm{d}u_t$$
$$+ c_c(K_{t-1} + R_t - E_tL_t),$$

from which it follows that

$$\frac{\partial C}{\partial R_t} = -(c_f + c_c) \int_{K_{t-1}+R_t-E_tL_t}^{\infty} F'(u_t)\,\mathrm{d}u_t + c_c,$$

$$\frac{\partial^2 C}{\partial R_t^2} = (c_f + c_c)F'(K_{t-1} + R_t - E_tL_t) > 0.$$

Setting the first derivative to zero implies that

$$P(u_t > E_tK_t = K_{t-1} + R_t - E_tL_t) = c_c/(c_f + c_c).$$

Parts (a)–(c) of the proposition follow directly from these results. $\square$

Proof of Proposition 2. The coefficients of K_{t+i+1} and its lags in Eq. (29) may be expressed as $1 - \frac{\gamma}{\beta}L + \frac{1}{\beta}L^2 = (1 - \lambda_1 L)(1 - \lambda_2 L)$, where L is the lag operator (used only in this proof and not to be confused with losses) and

$$\lambda_1 = \left[1 + \beta a + a - \sqrt{(1 + \beta a + a)^2 - 4\beta a^2}\right] \Big/ (2\beta a).$$

Thus, $0 \leqslant \lambda_1 < 1$ and $\lambda_2 = \frac{1}{\beta\lambda_1} > 1$. Note that $\frac{L}{1-\lambda_2 L} = \frac{-\beta\lambda_1}{1-\beta\lambda_1 L^{-1}}$ and multiply (29) throughout by this ratio. The left-hand side expression for the ratio cancels the second factor in the coefficient polynomial for K_{t+i+1}, whereas the right-hand side expression is applied to the terms in L_{t+i+1} and to the constant term. These last two operations lead to the sum of expectations and the constant term in Eq. (30). Eq. (31) follows by substituting (30) into (4). $\square$

Proof of Proposition 3. Use Eq. (33) to express the expectational terms in (31) as a linear combination of L_{t-1} and L_{t-2} in which the coefficients are infinite geometric sums involving powers of λ_1, β, m_1 and m_2. Applying standard closed-form expressions for these infinite sums, we arrive at (35), where

$$\delta_1 = \frac{(1 - \lambda_1)(2\cos\omega - \lambda_1\beta)}{1 - 2\lambda_1\beta\cos\omega + \lambda_1^2\beta^2},$$

$$\delta_2 = \frac{\lambda_1 - 1}{1 - 2\lambda_1\beta\cos\omega + \lambda_1^2\beta^2}.$$

Eq. (34) then follows from (4) and (35). $\square$

Proof of Proposition 4. The coefficients of the spectral regressions in Section 3.2. may be written as

$$
b_{RL}(\omega) = \frac{(1-\lambda_1)(1-\lambda_1\beta)[(1-\lambda_1)(1-\lambda_1\beta) + \lambda_1(1+\beta)(1-\cos\omega)]}{\left[(1-\lambda_1)^2 + 2\lambda_1(1-\cos\omega)\right]\left[(1-\lambda_1\beta)^2 + 2\lambda_1\beta(1-\cos\omega)\right]},
$$

$$
b_{\Delta KL}(\omega) = b_{RL}(\omega) - 1,
$$

$$
b_{KL}(\omega) = \frac{-\lambda_1\left\{\left[(1-\beta)^2 + 2\beta(1-\cos\omega)\right](1-\cos\omega)\lambda_1 + (1-\beta\cos\omega)(1-\lambda_1\beta)(1-\lambda_1)\right\}}{\left[(1-\lambda_1)^2 + 2\lambda_1(1-\cos\omega)\right]\left[(1-\lambda_1\beta)^2 + 2\lambda_1\beta(1-\cos\omega)\right]},
$$

$$
b_{IL}(\omega) = b_{KL}(\omega) + 1.
$$

It is immediately clear that $b_{RL}(\omega) \geqslant 0$ (hence $b_{\Delta KL}(\omega) \geqslant -1$) and that $b_{KL}(\omega) \leqslant 0$ (hence $b_{IL}(\omega) \leqslant 1$). It may also be shown that $b_{RL}(\omega) \leqslant 1$, hence $b_{\Delta KL}(\omega) \leqslant 0$. Note that it is possible that $b_{IL}(\omega) < 0$ if adjustment costs are high (λ_1 close to 1) and the cycle is long (ω close to 0). $\square$

Proof of Proposition 5. Eq. (39) may be expressed as

$$
\phi_{KL}(\omega) = \tan^{-1}\left[\frac{\mathrm{Im}(h_{KL}(\mathrm{e}^{-\mathrm{i}\omega}))}{\mathrm{Re}(h_{KL}(\mathrm{e}^{-\mathrm{i}\omega}))}\right].
$$

Because of the periodicity of the tan function (π), $\tan^{-1}$ is multi-valued and one must choose a solution according to the signs of $\mathrm{Re}(h_{KL}(\mathrm{e}^{-\mathrm{i}\omega}))$ and $\mathrm{Im}(h_{KL}(\mathrm{e}^{-\mathrm{i}\omega}))$. These signs are determined, respectively, by

$$
-\lambda_1\left[(1-2\beta\cos\omega+\beta^2)\lambda_1 + \beta(1-\beta\lambda_1)(1-\lambda_1)\right] < 0
$$

and

$$
\lambda_1\left[(1-2\beta\cos\omega+\beta^2)(1-\cos\omega)\lambda_1 + (1-\beta\cos\omega)(1-\beta\lambda_1)(1-\lambda_1)\right]\sin\omega > 0,
$$

so that $\pi/2 \leqslant \phi_{KL}(\omega) \leqslant \pi$. The argument of the $\tan^{-1}$ function may be written as

$$
A(\lambda_1) = -\frac{\left[(1-2\beta\cos\omega+\beta^2)\lambda_1 + \beta(1-\beta\lambda_1)(1-\lambda_1)\right]\sin\omega}{(1-2\beta\cos\omega+\beta^2)(1-\cos\omega)\lambda_1 + (1-\beta\cos\omega)(1-\beta\lambda_1)(1-\lambda_1)},
$$

which satisfies $A(0) = \beta\sin\omega/(1-\beta\cos\omega)$ and $A(1) = \sin\omega/(1-\cos\omega)$. We also have that

$$
\frac{\mathrm{d}A}{\mathrm{d}\lambda_1} = -\frac{(1-2\beta\cos\omega+\beta^2)(1-\beta)(1-\lambda_1^2\beta)\sin\omega}{\left[(1-2\beta\cos\omega+\beta^2)(1-\cos\omega)\lambda_1 + (1-\beta\cos\omega)(1-\beta\lambda_1)(1-\lambda_1)\right]^2},
$$

which is positive for $0 < \lambda_1 \leqslant 1$ and $0 < \omega < \pi$. This proves part (a) of the proposition. For part (b), we use three standard trigonometric identities,

$\sin 2\theta = 2 \sin \theta \cos \theta$, $\sin^2 \theta = (1 - \cos 2\theta)/2$, and $\cot^{-1} x = \pi/2 - \tan^{-1} x$. From the first two, with $\theta = \omega/2$, we obtain that

$$\frac{\sin \omega}{1 - \cos \omega} = \cot \frac{\omega}{2}.$$

Applying the third identity to this equation and using the fact that $\tan^{-1}$ is an odd function results in

$$\tan^{-1} \left[-\frac{\sin \omega}{1 - \cos \omega} \right] = \frac{\omega}{2} - \frac{\pi}{2} + \pi = \frac{\omega}{2} + \frac{\pi}{2},$$

where π must be added to the solution so that it satisfies $\pi/2 \leqslant \phi_{KL}(\omega) \leqslant \pi$. Finally,

$$-\frac{1}{\omega} \tan^{-1} \left[-\frac{\sin \omega}{1 - \cos \omega} \right] = -\frac{1}{2} - \frac{\pi}{2\omega} = -\frac{1}{2} - \frac{q}{4},$$

where we define $q \equiv 2\pi/\omega$. $\quad\square$

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